Lecture 7 summary:
Fundamentals of Reservoir Simulation

3. Fundamentals of Reservoir Simulation

How the simulator solves the complex fluid flow equations?

3.1 Solution Procedures

The basic conservation laws of reservoir simulation are the conservation of mass, momentum and energy.

Mass balance in a grid block is achieved by equating the accumulation of mass in the grid block with the difference between the mass leaving the grid block and the mass entering it.

The ability of the simulator to account for flow between grid blocks is what makes a simulator different from a reservoir engineering material balance program.

The advantage of using a simulator instead of a material balance program is that the simulation model can be expanded to include flow in one, two, or three spatial dimensions.

Momentum conservation is modeled using Darcy's Law.

Energy conservation is modeled by equations of state.

Fluid flow equations are a set of nonlinear differential equations that must be solved by computer.

The derivatives are replaced with finite differences, which are in turn derived from Taylor's series. Table 9 outlines this procedure.

The finite difference interval Δx along the *x*-axis is called the grid block length, and the finite difference interval Δt is called the time step.

Indices *i*, *j*, and *k* are used to label grid locations along the *x*, *y*, and *z* coordinate axes, respectively. Index *n* labels the present time level, so that *n*+1 represents a future time level.

If the finite difference representations of the partial derivatives are substituted into the original flow equations, the result is a set of equations that can be algebraically rearranged to form a set of equations that can be solved numerically. The solution of these equations is the job of the simulator.

The two most common solution procedures in use today are:

a) The fully implicit (Newton-Raphson) technique: all primary variables are calculated at the same time and at the new time level they are determined simultaneously.

The derivatives are stored in a matrix called the acceleration matrix or the Jacobian. The matrix equation is solved by matrix algebra to yield the changes to the primary unknown variables.

Table 9 Finite Difference Approximation

1. Formulate fluid flow equations, such as,

$$\frac{\partial}{\partial x}\left[\frac{Kk_r}{\mu B}\left(\frac{\partial P}{\partial x}\right)\right]+q_s\delta(x-x_0)=\frac{\partial}{\partial t}\left(\frac{\phi S}{B}\right)$$

2. Approximate derivatives with finite differences
   a. Discretize region into gridblocks Δx:

$$\frac{\partial P}{\partial x}\approx\frac{P_{i+1}-P_i}{x_{i+1}-x_i}\equiv\frac{\Delta P}{\Delta x}$$

   b. Discretize time into timesteps Δt:

$$\frac{\partial S}{\partial t}\approx\frac{S^{n+1}-S^n}{t^{n+1}-t^n}\equiv\frac{\Delta S}{\Delta t}$$

3. Numerically solve the resulting set of linear algebraic equations

These changes are added to the value of the primary unknown variables at the beginning of the iteration. If the changes are less than a specified tolerance, the iterative Newton-Raphson technique is considered complete and the simulator proceeds to the next time step.

The three primary unknown variables for an oil-water-gas system are oil-phase pressure, water saturation, and either gas saturation or solution gas-oil ratio.

b) Implicit pressure, explicit saturation (IMPES)

It is much like the Newton-Raphson technique except that flow coefficients are not updated in an iterative process.

By contrast, the IMPES procedure solves for pressure at the new time level using saturations at the old time level, and then uses the pressures at the new time level to explicitly calculate saturations at the new time level.

A variation of this technique is to iteratively substitute the new time level estimates of primary variables in the calculation of coefficients for the flow equations.

The iterative IMPES technique takes longer to run than the non iterative technique, but generates less material balance error.

The simulation program begins by reading input data and initializing the reservoir..

Once the primary variables are determined, the process can be repeated by updating the flow coefficients using the values of the primary variables at the new iteration level. This iterative process can improve material balance.

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When the solution of the fluid flow equations is complete, flow properties are updated and output files are created before the next time step calculation begins.

The fully implicit simulator can solve problems faster than IMPES techniques by taking significantly longer time steps.

In summary, a representation of the reservoir is quantified in the reservoir flow simulator. The representation is validated during the history matching process, and forecasts of reservoir performance are then made from the validated reservoir representation.

Figure 16 shows a flow chart for a typical simulator.

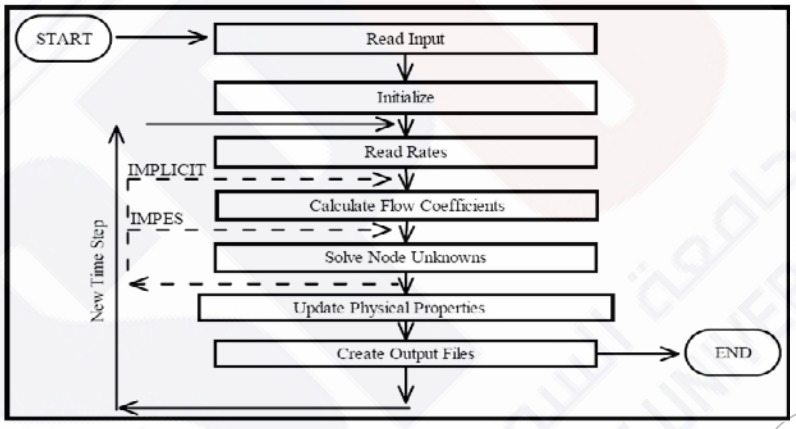

Figure 16. Typical simulator flow chart

3.2 Volume Integration and Discretization

The fluid flow equations are discretized using volume integration and finite difference techniques.

3.3 Transmissibility model

Flow between neighboring grid blocks is treated as a series application of Darcy's Law simulator.

A transmissibility term between two grid blocks is defined using the product of average values of relative permeability k_{rA} of phase A, absolute permeability K of each grid block at the interface, and cross-

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sectional area A_c of each grid block, divided by the product of the viscosity μ_A of phase A and the formation volume factor B_A in grid block.

3.4 Well Model

A well model in simulator is a variation of Darcy's Law which says that well flow rate is proportional to pressure change.

The relationship between flow rate Q_ℓ of phase ℓ and pressure change ΔP may be written as:

$$Q_\ell = PI\ \Delta P \text{ , ---------- (20)}$$

where the proportionality constant is called the productivity index (PI).

3.5 Solution Constraints

Rate Constraint Representation

In the rate constraint representation, well rates may be specified for injectors or producers.

Pressure Constraint Representation

In the pressure constraint representation, pressure differentials are used to calculate flow rates for injectors or producers.

GOR Constraints

Maximum (GOR_{max}) (WOR_{max}) can be entered by the user for each oil production well.

If GOR for the well exceeds GOR_{max}, then the completion interval (connection) with the highest GOR will be shut in. The procedure is repeated until GOR is less than GOR_{max} or until the well is shut in.

The WOR is defined as total water production for all active well completion intervals during the time step divided by total oil production for all active well completion intervals during the time step.

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If WOR for the well exceeds WOR_{max}, then the completion interval (connection) with the highest WOR will be shut in. The procedure is repeated until WOR is less than WOR_{max} or until the well is shut in.

Fluid Withdrawal Constraints

Fluid withdrawal can be constrained for primary phases as follows:

A. A minimum production rate QWMIN can be specified.

B. A maximum production rate QWMAX can be specified.

Primary phases subject to fluid production constraints are oil, water, natural gas, and total fluid.

Fluid Injection Constraints

Fluid injection using explicit pressure controlled injection wells can be constrained for primary phases as follows:

A. A minimum injection rate QWMIN can be specified.

B. A maximum injection rate QWMAX can be specified.

Primary phases subject to fluid injection constraints are water and natural gas.